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Methods for the sequencing of sheet metal bending operations

J. R. DUFLOU†*, D. VAN OUDHEUSDEN‡, J.-P. KRUTH§,
and D. CATTRYSSE‡

The sequencing of part set-ups, in the context of design verification or process planning activities for sheet metal bending operations, is a rather complex combinatorial problem. The verification of the feasibility and the acceptability of a single bend set-up requires CPU-time consuming operations, e.g. collision checking and a manipulation requirement analysis. For more complex parts, and thus increased numbers of bends, exhaustive search methods can therefore not be applied in a time-economic way. Although the identification of an optimal sequence may not always be possible, a number of techniques can be applied to significantly downscale the problem size. Several complementary approaches have been worked out to proceed in identifying near-optimum feasible bend sequences. Constraint solving and branch-and-bound techniques are used to identify interesting potential solutions. The reported branch-and-bound search method is characterized by a dynamically updated penalty system, that reflects the manufacturing knowledge obtained through analysis of partial sequences. A case study is used to illustrate the effectiveness of the applied search procedures.

1. Nomenclature

C	incidence vector
CNC	computer numerical controlled
n	number of bends in a part configuration
p_x	penalty corresponding to criterion x
PCM	precedence constraint matrix
Rx	rule matrix for rule x
S	bend sequence
TSP	travelling salesman problem

2. Problem statement

Sheet metal bending by means of CNC or conventional press brakes is a manufacturing process that requires constant operator involvement. In the case of CNC bending, a workpiece is typically formed in a number of consecutive steps, performed as a single, uninterrupted sequence. For small and medium lot sizes, the machine set-

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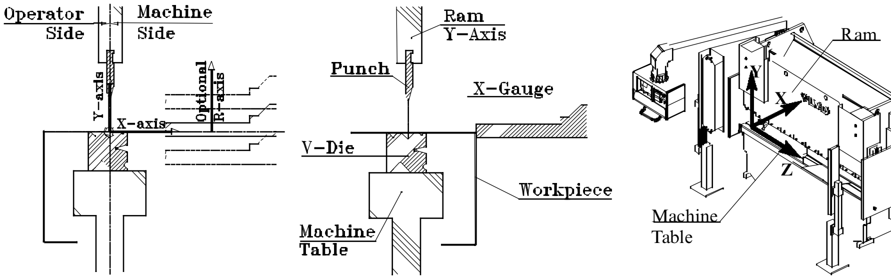


A B

Figure 1. Workpiece orientation options per bend set-up relative to machine geometry.

up time (tooling) and the workpiece manipulation in between consecutive bending operations are the most important factors influencing the time in process and thus the cost effectiveness of this manufacturing stage. Process plan optimization is required in order to minimize the total production time.

Process planning for sheet metal bending involves a series of activities, the most crucial ones being: technological verification (minimum and maximum dimensions, allowed stresses/forces, etc.), identification of suitable gauging opportunities, collision checking, manipulation evaluation and tolerance verification (Shpitalni and Saddan 1994, De Vin *et al.* 1995, Duflou 1996). The final aim is, after comparison of possible alternatives, to identify a sequence of part set-ups that requires the least total tooling and actual production time.

The domain of possible sequences contains in principle all $n!$ permutations, n being the number of bends in a given part. For every set-up a choice needs to be made between two possible orientations of the workpiece, as illustrated in figure 1A, B, leading to a total of $2^n n!$ sequences. A typical three-dimensional sheet metal part easily contains 12 or more bends. The aim to reduce the total number of parts in an up-to-date design for manufacturing approach created a trend towards more complex parts with increased number of bends: $n = 20$ is in consequence a realistic order of magnitude to characterize the problem.

A more efficient representation scheme, based on bend combinations rather than permutations, allows one significantly to reduce the maximum amount of relevant information, and thus to improve the space efficiency of solving methods (Duflou 1997). The basis for this representation scheme is the recognition of the fact that, for an intermediate state of a workpiece, the temporary shape of the part is defined by the combination of all bends already performed and is independent of the sequence in which these were executed.

Still, exhaustive searches in this combination space lead to unacceptable response times in cases where higher numbers of bends are involved. In such cases a more intelligent pre-identification of potentially interesting sequences is mandatory in order to run the different test and evaluation modules of a process planning system in a time-efficient way.

For small batch sizes a process planning task is often not aimed at finding the optimum sequence solution. In some cases, the objective is even limited to the identification of a single feasible solution for a given part design, machine and tool set. Especially in job shop environments, process planning could easily become a more resource-consuming activity than the actual shop floor production. The process planning procedure itself should therefore allow one to provide a near optimum solution within a reasonable time-frame. If the sequencing algorithm needs to be implemented in the machine controller, the acceptable processing time becomes an even more critical constraint: the controller occupancy for process planning purposes has a direct effect on the productiveness of the CNC press brake. Acceptable response times should be of an order of magnitude of seconds for simple parts, or minutes for more complex geometries.

In a variant approach, process plans could be derived from existing solutions for similar part topologies, using group technology-based data retrieval. However, in contrast with manufacturing methods based on material removal, sheet metal processing typically allows a large variation of possible part configurations, limiting this approach to specific cases where the number of typical part topologies can be expected to be rather static. Additionally, the strong dependency between the scale of part details on the one hand, and the applicability of a given set of tools in a predefined sequence on the other, also obstructs the effective use of variant process planning procedures.

When developing an automatic process planning system based on a generative principle, the combinatorial scale of the search domain can be contained by applying techniques such as those often implicitly used by experienced manual process planners. For a given machine and a limited series of available tools, skilled process planners often identify a number of geometric constraints that allow a significant reduction of the problem size (Geiger *et al.* 1992, Duflou and Kruth 1997). These constraints, further referred to as hard constraints, can be supplemented with information obtained from 'good practice' heuristic rules (De Vin *et al.* 1995, Radin and Shpitalni 1996, Duflou and Kruth 1997). Both types of information are applied in the procedures described hereafter. A first method, described in section 2, is limited to the systematic identification and inference of constraints related to the relative sequence of bends. In the method presented in section 3, additional information related to preferences for sub-sequences can be incorporated. The quantification obtained from this last method allows optimization using branch-and-bound techniques.

3. Precedence constraint-based method

Limited information related to the relative positions of two or more specific bends in a sequence allows the significant reduction of the number of potential solutions to be considered and the total number of bend simulations required for systematic solution comparison. The extent to which the maximum number of bend evaluations is affected by the identification of precedence constraints is illustrated in figure 2.

This leads to the seemingly contradicting observation that more complex part geometries often define less complex search problems due to the larger number of constraints linked to them. In extreme cases, constraint solving for a preselected press brake and a set of tools could lead to a single feasible solution. In other cases, early detection of contradicting precedence constraints could help to identify part designs that are not well adjusted to bending as a manufacturing process, without requiring exhaustive verification.

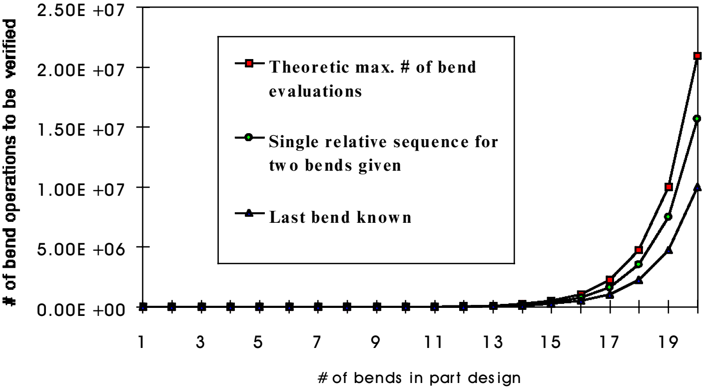


Figure 2. Problem size reduction through relative position identification.

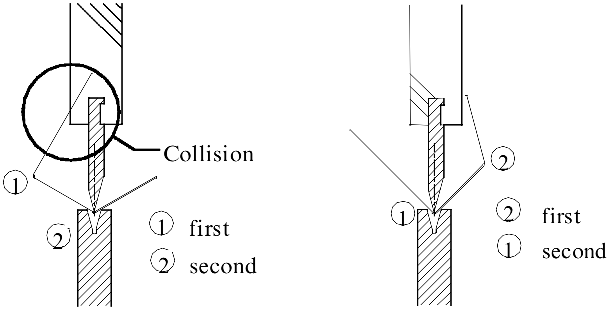


Figure 3. Proper relative sequence position (right) derived from preprocessing stage analysis for a Γ-detail.

3.1. *Precedence constraint identification*

Analysis of typical part details allows the identification of precedence constraints for a given machine geometry, equipped with a preselected set of punch and die tools. This preprocessing stage of the actual sequencing procedure is based on bending simulation for a limited number of isolated flanges and interconnecting bends. The analysis of a typical Γ-detail, as shown in figure 3, e.g. frequently leads to the identification of hard precedence constraints. Non-compliance with such a constraint would lead to a collision between the workpiece and the punch or machine.

A typical example of a detail that leads to the identification of a hard precedence constraint is a two-stage hemmed edge, as illustrated in figure 4: in a first operation a bend angle $\geqslant 90^\circ$ is executed, followed by a finishing step with a hammer tool that completes a bend over an angle of 180°. It is clear that, although no immediate succession is required, the sequence of both operations cannot be reversed.

Besides hard constraints derived from topological details, a number of empirical rules can also provide precedence prescriptions. These rules are based on observations of the way experienced process planners analyse workpieces to be manufactured.

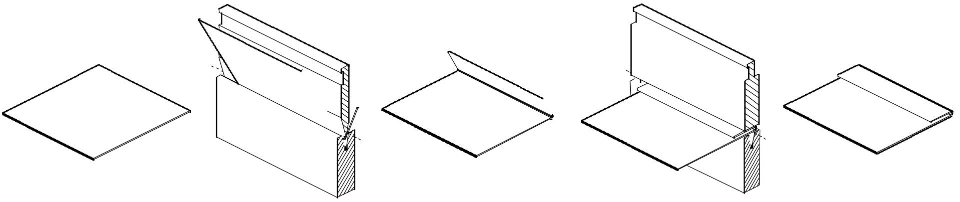


Figure 4. Two-stage hemming process.

Some examples are as follows

(1) 'Bends that have a major influence on the all-over shape of the part ('shape defining' bends) are best performed after non-shape-defining bends'.
(2) 'Longer bends are preferably formed after significantly shorter ones'.
(3) '90° bends are best performed before other bends'

In contrast with the precedence prescriptions based on geometric constraints, these rules have a fuzzy character: the degree of applicability of the rules can vary with the part geometry, and the relative importance of the rules is not clearly defined.

3.2. *Constraint representation*

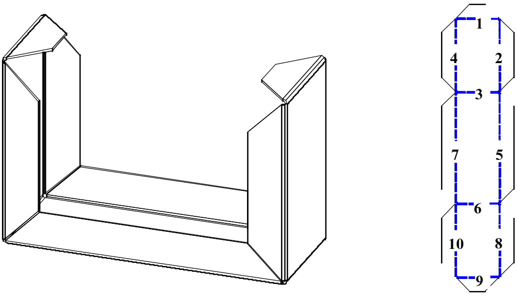
Precedence constraints can easily be represented in a directed graph. The relative importance attached to the hard constraints and the results of heuristic rule application can be indicated by using a matrix notation with different priority levels, as illustrated in figure 5. The row and column numbers correspond with the bend ID numbers, while the matrix cells contain the information related to the priority level of the constraints ($0 =$ no constraint, $1 =$ hard constraint, $>1 =$ heuristic constraint). A cell value $a_{ij} > 0$ thus refers to a constraint indicating that bend i should precede bend j. Using increasing integers for additional precedence constraints, based on lower priority rules, allows the addition of information to an existing precedence constraint matrix (PCM). In cases where different rules lead to multiple precedence constraints for a given couple of bends, only the highest priority rule is retained.

For the part as described in figure 5, the detail analysis reveals two hard constraints, i.e. precedes (1, 3) and precedes (9, 6). Deriving relative sequence information from heuristic rules 1 and 2, as listed in the previous paragraph, with corresponding cell values 2 and 3, would result in the matrix representation given in figure 5. Cell a_{12}, e.g. was set equal to 3 as, according to heuristic rule 2 (priority level 3), bend 1 should preferably be bent before the significantly longer bend 2.

3.3. *Consistency verification*

The resulting precedence constraint matrix (A) can be tested for possibly conflicting information. Symmetrical precedence constraints (e.g. figure 5: a_{23} and a_{32}) need to be eliminated based on the indicated priority levels of the corresponding heuristic rules. For the example of figure 5 this leads to the matrix shown in figure 6.

Where symmetrical hard constraints are detected, either an adjusted tool/machine preselection or a part redesign is mandatory. Transitivity of the precedence relationship could lead to reflexive constraints. Closed loops in the directed graph therefore need to be identified and eliminated. In the inference method described in



	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	■	3	1	3	2	2	2	3	0	3
2	0	■	2	0	2	2	2	0	0	0
3	0	3	■	3	3	0	3	3	0	3
4	0	0	2	■	2	2	2	0	0	0
5	0	0	0	0	■	0	0	0	0	0
6	0	3	0	3	3	■	3	3	0	3
7	0	0	0	0	0	0	■	0	0	0
8	0	0	2	0	2	2	2	■	0	0
9	0	3	2	3	2	1	2	3	■	3
10	0	0	2	0	2	2	2	0	0	■

Figure 5. Sample part and derived precedence constraint matrix A.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	■	3	1	3	2	2	2	3	0	3
2	0	■	2	0	2	2	2	0	0	0
3	0	0	■	0	3	0	3	0	0	0
4	0	0	2	■	2	2	2	0	0	0
5	0	0	0	0	■	0	0	0	0	0
6	0	0	0	0	3	■	3	0	0	0
7	0	0	0	0	0	0	■	0	0	0
8	0	0	2	0	2	2	2	■	0	0
9	0	3	2	3	2	1	2	3	■	3
10	0	0	2	0	2	2	2	0	0	■
C	0	2	6	2	8	6	8	2	0	2

Figure 6. Precedence constraint matrix A after anti-symmetry verification, and incidence vector C.

the next paragraph, transitivity conflicts are automatically detected. A preliminary identification of the occurrence of loops in the graph is therefore not required.

3.4. *Constraint solving*

After elimination of lower priority conflicting symmetrical constraints, the task to identify one or more sequences, that meet the conditions imposed by the remaining constraints, can be initiated. In general, there will be several possible first bends available to choose from. These can be identified systematically through analysis of the precedence constraint matrix A. An incidence vector (C) can be calculated:

$$c_j = \sum_{i=1}^{n} \delta(a_{ij}) \quad \text{for} \quad a_{ij} \in A$$

$i, j \in [1, n]$
n: number of bends

Acceptable first bends j are identified by $c_j = 0$. The corresponding column a_{xj} and row a_{jy} vectors can be removed from the PCM, followed by a recalculation of the incidence vector C. Based on the characteristic that acyclic-directed graphs contain at least one vertex with in-degree 0, the occurrence of one or more closed loops will be recognized when, after the elimination of already sequenced bends and recalculation of the incidence vector, no bends can be identified as available for further sequencing (for all remaining bends j: $c_j > 0$). This situation implies that all remaining bends are involved in closed loops of precedence constraints. Starting from a random selected bend, a series of consecutive edges in the corresponding graph can be followed until one of the already encountered bends reoccurs. Repetitive application of the transitivity characteristic would result in a reflexivity conflict: $a_{ij} > 0$, $a_{jk} > 0, \ldots, a_{li} > 0 \Rightarrow a_{ii} > 0$.

Two cases should be distinguished.

- $a_{ij} = 1$ and $a_{jk} = 1$ and $\ldots$ $a_{li} = 1$. A hard constraint $a_{ii} = 1$ is derived from the transitivity characteristic. The problem, as described by the geometry of the part and the preselected tooling, cannot be solved. Either or both needs to be adjusted.
- $a_{ij} > 1$ or $a_{jk} > 1$ or $\ldots$ or $a_{li} > 1$. At least one of the constraints was obtained from a non-imperative, heuristic rule and can possibly be eliminated, without necessarily causing geometric interference as a result. In this case, the constraint with the lowest priority (highest value a_{ij}) will be eliminated and the incidence vector can be recalculated starting from the adjusted precedence matrix. This procedure may need to be applied repeatedly in order to be able to identify one or more bends with order 0 for further sequencing.

Solution of the sample problem described in figures 5 and 6, with the procedure as formulated above, results in a suggested sequence $S = [1, 9, 2, 4, 8, 10, 3, 6, 5, 7]$. Multiple solutions comply with the constraints of the formulated problem, as could already be logically concluded from the double symmetrical part layout. Alternatives are recognizable by observing the multiple 0-values emerging in the incidence vector C at several stages in the procedure. The sequence S allows collision-free manufacturing of the part. However, a total of 10 separate bend operations are prescribed, of which the last two require a dedicated tool set-up using horn-punches to avoid part–tool interference. In total, three different tool sets need to be mounted on the machine in order to be able to execute the obtained sequence.

3.5. *Limitations*

The method as described above allows systematic representation and inference of constraints indicating relative positions of bends in a sequence only. Information concerning the preferred consecutive occurrence of a series of bends cannot be easily incorporated in a precedence constraint-based system. For example, a rule indicating that bends of equal length should preferably be executed as a continuous series cannot be taken into account by the method described above.

Other important shortcomings of the precedence constraint solver are the incapability to make optimal use of the opportunities offered by parts containing bends that may be produced simultaneously in a single bending stroke. The topology of some parts indeed allows the combination of two or more bends: equal bend angles and bend lines that are collinear in an unfolded status in principle correspond to bend operations that can be performed in a single part set-up. Examples are bends 4, 7, 10 and 2, 5, 8 in figure 5. Although an adjusted tool set-up may be required, such reduction of the total number of bend operations often allows the significant reduction of the total processing time for larger batch sizes. When two flanges are interconnected by multiple bends, combining the corresponding bend operations becomes compulsory. An example of such a case is shown in figure 7.

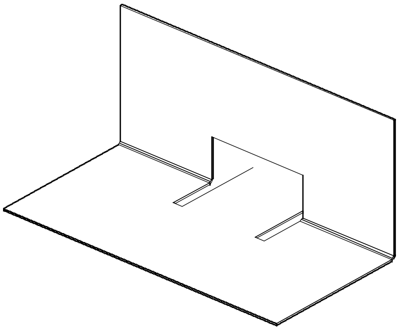


Figure 7. Multiple bend connection between flanges: compulsory combined bends.

Information related to the optional or compulsory combinability of bends cannot be represented in a single directed graph and the related precedence constraint matrix. An improved system should, additionally, be capable of handling situations where the combination of bend operations can be disturbed by the preceding execution of bends that affect the collinearity of the bend lines (e.g. bends 3 and 6 in figure 5).

The relative importance attached to the respective heuristic rules applied in a sequencing exercise is of major influence due to the non-cumulative representation scheme and the systematic elimination of lower priority constraints in cases where conflicting precedence prescriptions are generated. This approach limits the differentiation made in the precedence relationship to a few priority levels. Indeed, although a number of heuristic rules may reinforce the preference for a certain precedence relationship, the retained information only reflects the triggering of the rule with the highest priority level.

The remaining part of this paper is dedicated to the description and demonstration of a more refined, travelling salesperson (TSP)-based method, that overcomes the limitations of the precedence constraint method as presented above.

4. TSP-based method

The description of a bending operation as a transition between two intermediate states of a workpiece, with a number of already formed bends $i - 1$ and i $(1 \leqslant i \leqslant n)$, respectively, logically leads to a graph representation with the different states of the workpiece represented as nodes and the bend operations as arcs. The problem under consideration could, however, also be described as the identification of a (near) optimum path along a series of n bend operations, with the restriction that each bend should be included in the path exactly once. This last approach implies the introduction of a second graph in which the nodes represent bend operations and the arcs stand for the transition between consecutive bend set-ups.

4.1. *TSP description*

If every bend operation can be represented as a node in a graph, the simple introduction of a single, dummy, combined start and end node allows the description of the problem as a TSP. In contrast with a conventional TSP, however, the cost values linked to arcs in the directed graph are not static: a good cost estimate can only be made based on the knowledge of all previously passed nodes in the path under construction, and thus needs to be calculated dynamically during the search procedure.

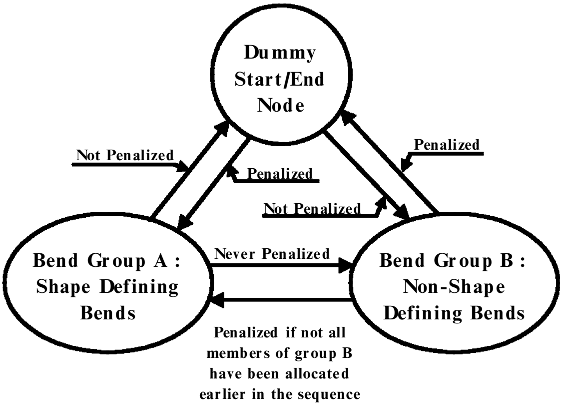


Figure 8. Penalty system corresponding to a 'two-group' heuristic rule: e.g. 'shape-defining beads are best performed after non-shape-defining ones'.

This can be illustrated by considering some of the heuristic rules and the consequences of the path history for the penalty values derived from these rules. As illustrated in figure 8, the answer to the question whether the choice for a certain arc complies with a given rule, depends on the bends already included in the partial sequence.

Another example is hard constraints imposing compulsory relative sequences: if *Precedes* (a, b) is a hard constraint, then any attempt to insert bend b in a sequence, before bend a has been included, would necessarily lead to a collision later in the sequence and should therefore receive a cost value $= \infty$.

In conclusion, the penalty matrix that corresponds to the directed graph should be constructed dynamically: state-dependent information should be updated and included in the penalty matrix just before every bend selection step. Knowledge that is independent of the composition of a partial sequence can be inserted in an initialization phase.

4.2. *Penalty system and objective function*

The comments related to process analysis formulated in section 2 remain valid: a detailed collision detection, manipulation requirement analysis and tolerance verification are required to evaluate the applicability of a bending sequence. These time-consuming evaluation steps can only be considered when an appropriate preselection of a possibly feasible (partial) sequence has been identified. The penalty values used for the TSP formulation should therefore be based on a limited part preprocessing effort, based on generic heuristic rules, rather than on detailed process simulation.

The hard constraint and rule information, identified as input for the precedence constraint solver in section 3, can be entirely recuperated as a first step in the penalty definition: the priority levels are replaced by predetermined penalty values, thus indicating the relative importance attached to the respective rules. The preferences derived from an applied generic rule can be saved as a static, binary $n * n$-matrix. For a given rule X the related rule matrix Rx contains penalty information based on a two-by-two comparison of all bends: $Rx_{ij} = 1$ if bend j should precede i according to rule X, otherwise $Rx_{ij} = 0$. The order of magnitude of the effort to construct these matrices is thus $O(n^2)$.

Minimum bend penalty	p_b
Penalty for hard constraint violation	p_{hc}
Penalty for heuristic rule violation	p_{h1}, p_{h2}, p_{h3},é
Penalty for continuous series violation	p_c
Penalty for optional combinable bend violation	p_{cb}
Penalty for compulsory combined bend violation	p_{ccb}

Figure 9. Penalty overview.

In order to make the penalty system reflect the total effort required to produce a part, a minimum penalty per bend operation p_b was introduced. This also allows the systematic penalization of the neglect of combinable bends, or the execution of one or more combinability disturbing bends before completion of a series of combinable bends. The non-simultaneous performance of a series of compulsory combined bends is equivalent to an unwanted three-dimensional deformation of the workpiece and will normally lead to the rejection of the final part. The corresponding penalty value p_ccb should be chosen accordingly. Additionally, a penalty p_c was introduced for the interruption of preferred continuous bend series. Systematic consultation of the rules matrices Rx for the applied heuristic rules, before selection of a bend operation, allows the introduction of penalties p_hx where appropriate.

The penalties, an overview of which is listed in figure 9, are applied in a cumulative way.

4.3. *Solving procedure*

Considering the scale of typical problems and the frequent need for quick identification of a non-optimized feasible solution, a depth first search approach was preferred.

The branch-and-bound procedure developed for solving the TSP was inspired by the algorithm described by Little *et al.* (1963). A major adjustment is the systematic path development starting from the completely formed part, working backwards towards the unfolded blank. Adding bend operations to a single end of the sequence under construction offers the advantage of allowing systematic updating of the penalty matrix. Working backwards is a more efficient procedure when including collision verification for partial sequences: first bends never result in collisions, while it is a common phenomenon that only a very limited number of bends can be executed as the last operation in the process plan. In general, collisions are harder to avoid in intermediate states where the part is already formed into a more compact shape, de facto towards the end of a bending sequence.

An overview of the search procedure described in the next paragraphs is given in the flowchart of figure 14. The sample part described in figure 5 is used to illustrate the consecutive procedure steps. For this purpose, the hard constraints identified in section 2, as summarized in PCM A of figure 5, were taken into account. The following heuristic rules were applied.

(1) ‘Shape-defining bends are best performed after non-shape-defining bends’ (weight 4).
(2) ‘Bends are preferably performed from the outer edges of the part towards the central flange’ (weight 2).
(3) ‘Longer bends are preferably formed after significantly shorter ones’ (weight 1).

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	■	0	0	0	0	0	0	0	0	0	-
2	0	■	0	0	0	0	0	0	0	0	17
3	-	0	■	0	0	0	0	0	0	0	1
4	0	0	0	■	0	0	0	0	0	0	17
5	0	0	0	0	■	0	0	0	0	0	10
6	0	0	0	0	0	■	0	0	-	0	1
7	0	0	0	0	0	0	■	0	0	0	10
8	0	0	0	0	0	0	0	■	0	0	17
9	0	0	0	0	0	0	0	0	■	0	-
10	0	0	0	0	0	0	0	0	0	■	17

Figure 10. Initialized penalty matrix for sample parts (figure 5).

4.3.1. *Penalty matrix initialization*

Based on the hard precedence constraints a number of arcs can already be eliminated from the graph in a preliminary stage. The corresponding cells in the $(n*(n+1))$-penalty matrix *P*, shown in figure 10, reflect a penalty value ∞ (–).

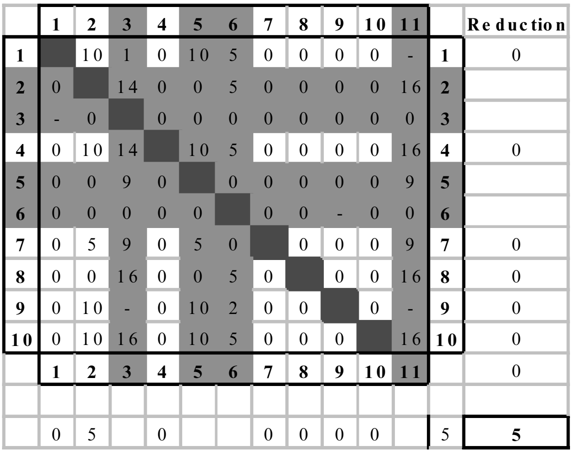
The additional column $(n+1)$ in the penalty matrix corresponds to the dummy start and end node. This column is initialized based on the hard constraints, the identified combinable and obstructing bends, the preferred continuous series and the information saved in the heuristic rule matrices *Rx*.

For cell $a_{1\ 11}$, e.g. the penalty value was obtained based on hard constraint considerations: bend 1 is not acceptable as the last bend in the sequence, as this would imply that, e.g. bend 3 would necessarily proceed bend 1, a relative sequence that will finally result in a collision. This information can be derived from the hard constraints saved in the PCM developed in section 2. To reflect this hard constraint violation, the corresponding penalty value was set at ∞ (–).

For cell $a_{2\ 11}$, a penalty value of 17 was accumulated: the penalty breakdown reflects the disturbance of a series of combinable bends and the violation of the applied heuristic rules $(+4+2+1)$. The combination of bends 2, 5 and 8 in a single bend operation cannot be maintained if bend 2 would be chosen as the last bend in the sequence: bends 3 and 6 would in that case be allocated earlier in the sequence and disturb the collinearity of the group of combinable bends (penalty + 10). The selection of bend 2 at this stage also does not comply with any of the three applied heuristic rules (penalty $+4+2+1$).

4.3.2. *Matrix reduction*

Penalties that cannot be avoided when continuing from an intermediate state can be eliminated by reducing the columns and rows in the penalty matrix that have a minimum penalty value >0. After every bend selection, the penalty matrix is updated based on the new intermediate state of the workpiece. This requires detailed analysis of the partial sequence, and the columns and rows corresponding to the not yet allocated bends in the PCM and the rule matrices R_x. The updated penalty matrix is then scanned for reduction opportunities. The sum of all column and row reduction values is used to augment the lower bound for the sequence under construction.



	1	2	3	4	5	6	7	8	9	10	11		Reduction
1		10	1	0	10	5	0	0	0	0	-	1	0
2	0		14	0	0	5	0	0	0	0	16	2	
3	-	0		0	0	0	0	0	0	0	0	3	
4	0	10	14		10	5	0	0	0	0	16	4	0
5	0	0	9	0		0	0	0	0	0	9	5	
6	0	0	0	0	0		0	0	-	0	0	6	
7	0	5	9	0	5	0		0	0	0	9	7	0
8	0	0	16	0	0	5	0		0	0	16	8	0
9	0	10	-	0	10	2	0	0		0	-	9	0
10	0	10	16	0	10	5	0	0	0		16	10	0
	1	2	3	4	5	6	7	8	9	10	11		0
	0	5		0			0	0	0	0		5	**5**

Figure 11. Matrix reduction: penalty matrix after fifth reduction for sample part, last allocated bend: 2.

4.3.3. *Operation selection*

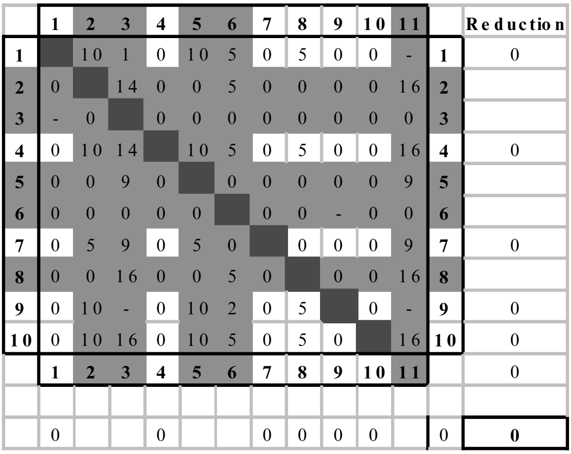
After reduction, the 0-penalty cells in the column of the last allocated bend operation indicate interesting choices for the selection of the next operation. Comparison of secondary, conditional penalties (Little *et al.* 1963) helps to distinguish between multiple candidates: operations with a higher secondary penalty are allocated first. The secondary penalty is used to calculate a lower bound for the alternative solutions branching off from the previously selected bend operation.

Unless intermediate collision verification would result in early backtracking, the matrix reduction and selection procedures will be repeated *n* times, until a first complete path, starting and ending in the dummy node, has been identified.

4.3.4. *Bounds and backtracking*

The reduction values accumulated during this repeated procedure provide an upper bound that allows systematic comparison for optimization purposes. Figure 13 shows the partial branch-and-bound tree for the sample part until the sixth reduction (corresponding to figure 12).

When one or more feasible solutions have already been identified, the lower bound developed during the reduction procedures needs to be compared to the best solution found so far (upper bound). Branches with lower bounds exceeding this upper bound should in principle not be further investigated. At this stage, the heuristic nature of some of the applied penalties should, however, be taken into account. The evaluation of a sequence based on a rule-based penalty system does not eliminate the need for a detailed analysis, including a collision verification and ergonomic evaluation. It is not unimaginable that such detailed simulation would result in an adjusted ranking of the suitability of different feasible bending sequences. Upper bounds should therefore not be treated as strict reject criteria, but merely serve to indicate an estimated degree of complexity of the solutions already identified. The likeliness that a partial sequence that exceeds the bound could still lead to a more suitable process plan decreases as the accumulated penalty value increases further. The uncertainty margin that would be allowed in order to guarantee that



	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**		**Reduction**
1		10	1	0	10	5	0	5	0	0	-	**1**	0
2	0		14	0	0	5	0	0	0	0	16	**2**	
3	-	0		0	0	0	0	0	0	0	0	**3**	
4	0	10	14		10	5	0	5	0	0	16	**4**	0
5	0	0	9	0		0	0	0	0	0	9	**5**	
6	0	0	0	0	0		0	0	-	0	0	**6**	
7	0	5	9	0	5	0		0	0	0	9	**7**	0
8	0	0	16	0	0	5	0		0	0	16	**8**	
9	0	10	-	0	10	2	0	5		0	-	**9**	0
10	0	10	16	0	10	5	0	5	0		16	**10**	0
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**		0
	0			0			0	0	0	0		0	**0**

Figure 12. Matrix reduction: penalty matrix after sixth reduction for sample part, last allocated bend: 8, next bend selection: 7.

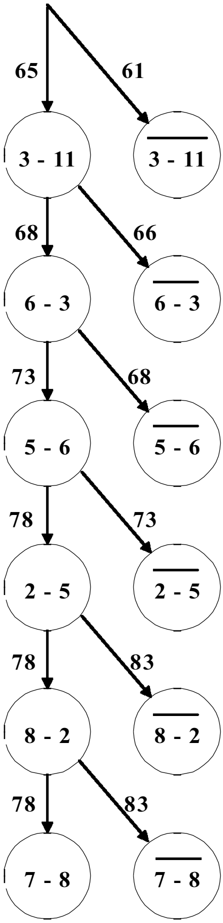


Figure 13. Branch-and-bound tree until sixth reduction, with indication of lower bounds.

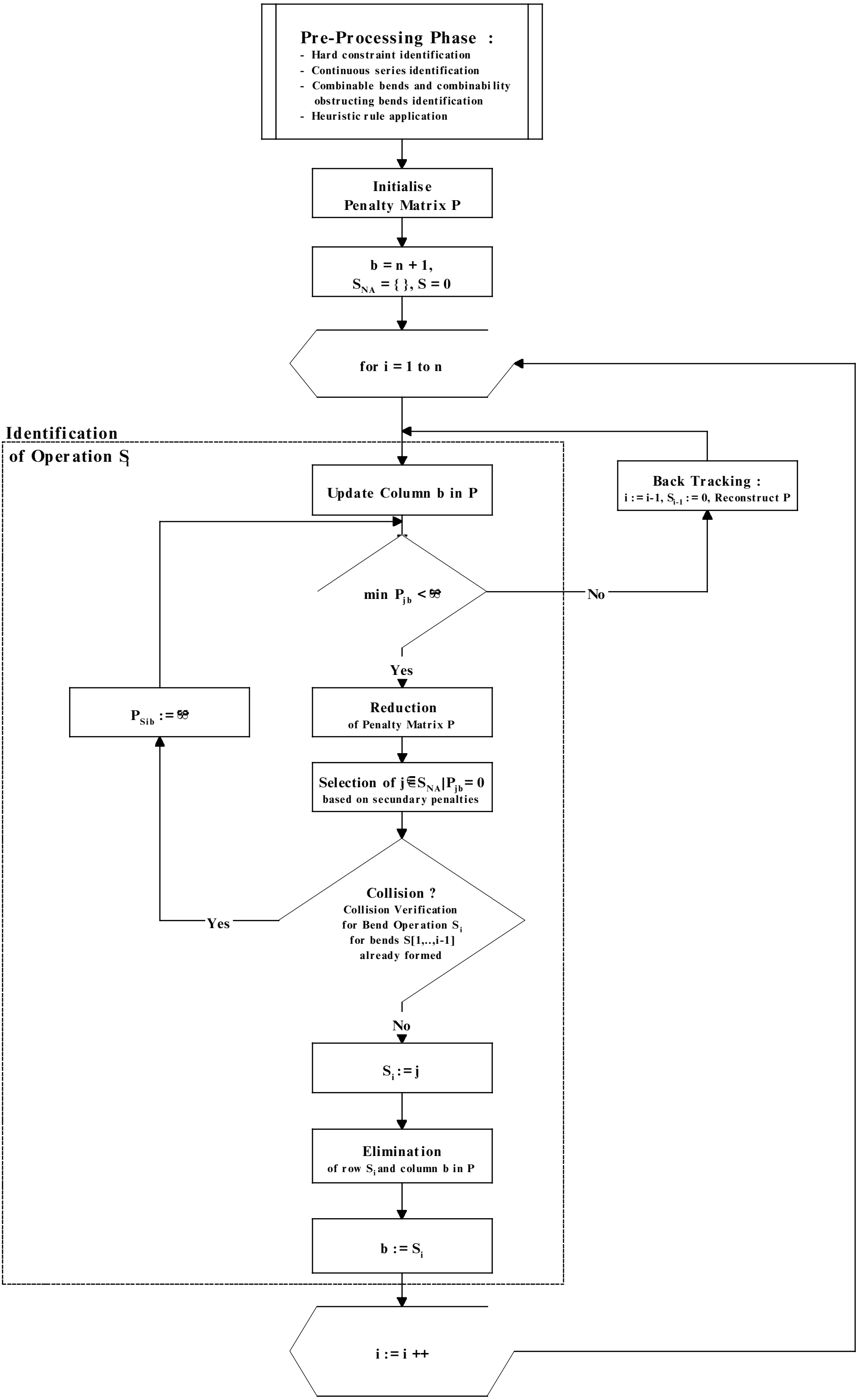


Figure 14. Overview search procedure first solution.

no potentially better optimized solutions would be eliminated, can be chosen in function of the acceptable total processing time.

Solving the sample part sequencing problem resulted in an initial sequence and penalty breakdown as listed in figures 15 and 16. The obtained solution corresponds well with the solutions generated by experienced process planners. The theoretic minimum of six operations to finish the part is achieved, thus respecting both series of combinable bends and allocating the obstructing bends after the combined series. The number of required tool sets is limited to two.

Further branch-and-bound optimization led to the identification of a number of solutions with a slightly lower total accumulated penalty. When taking the part symmetry into account, these solutions with minimal penalty value are identical. One of the obtained sequences and the corresponding penalty breakdown are listed in figures 17 and 18.

When evaluating the tool set-up and manipulation requirements, the solution proves to be of similar quality as the initial sequence: the tooling is more flexible as only one of the required tool sets requires a punch of limited length. The positioning of the workpiece when performing bends 1 and 9 requires more precise part control in order to be able to use the same punch as required for bends 3 and 6. For larger batch sizes, the initial solution would be preferred in order to optimize the actual production, while for a small series the tool set-up time could be minimized by opting for the minimal penalty sequence.

5. Case study

In order to test the effectiveness of the described methods for industrial application, a series of case studies was performed. For commonly encountered three-dimensional parts, the precedence constraint inference engine provided fast output of a near-optimum quality. More complex part topologies, however, often require additional information that can be contained in the TSP-based method.

In this section, a very complex case of a part composed of 31 flanges/30 bends is presented. Rather than aiming for a fully optimized solution, the purpose of this case study was to evaluate the initial depth first efficiency of the proposed TSP-based method. The related practical question was whether the developed procedures would allow the performance of fast automatic process planning for the manufacture of parts of this degree of complexity based on the first identified feasible solution.

5.1. *Part description*

Workpiece: stainless steel wall cover end component
Number of bends: 30
Number of flanges: 31
Number of nodes in graph: 1.07E09
Theoretic maximum number of bend evaluations: 3.22E10

5.2. *Constraints and applied heuristics*

The following workpiece characteristics were obtained as results of a preprocessing analysis of the part configuration.

Number of combinable bends: 2 groups of 5 bends.
group 1: (1, 3, 5, 27, 29), group 2: (2, 4, 6, 28, 30)

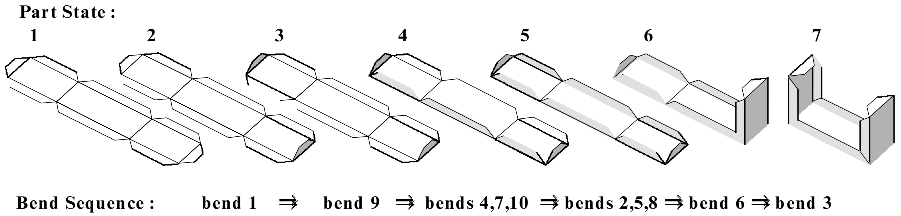


Figure 15. Sample part: first identified feasible sequence.

Rule	# of Violations	Penalty Weight	Penalty
Bend Minimum	6	10	60
Combinable Bends	0	10	0
Continuous Series	2	3	6
Heurist. Rule 1	2	4	8
Heurist. Rule 2	0	2	0
Heurist. Rule 3	4	1	4
		Total	**78**

Figure 16. Sample part: penalty breakdown for first identified feasible sequence.

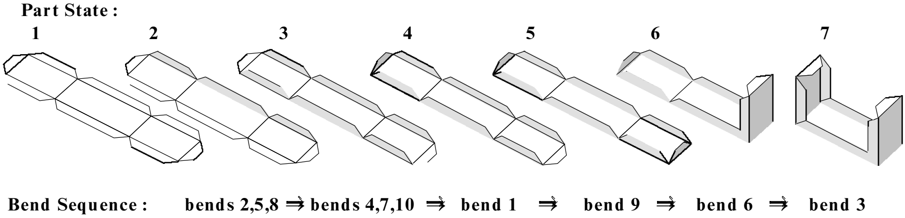


Figure 17. Sample part: minimal penalty sequence.

Rule	# of Violations	Penalty Weight	Penalty
Bend Minimum	6	10	60
Combinable Bends	0	10	0
Continuous Series	1	3	3
Heurist. Rule 1	2	4	8
Heurist. Rule 2	0	2	0
Heurist. Rule 3	6	1	6
		Total	**77**

Figure 18. Sample part: penalty breakdown for minimal penalty sequence.

Number of obstructing bends:
  for group 1: 4; (7, 8, 23, 24)
  for group 2: 9; (2, 4, 6, 7, 8, 23, 24, 28, 30)
Number of identified hard constraints: 5; *Precedes* (9, 8), (13, 12), (17, 16), (21, 20), (25, 24)

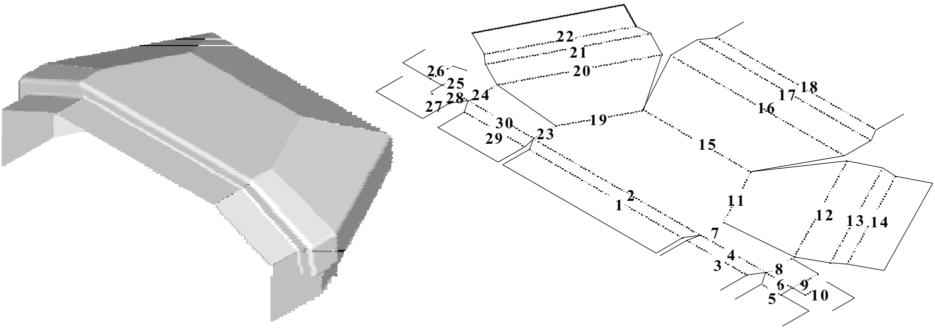


Figure 19. Sample part: folded state and unfolded part layout.

Identified continuous series: 10 series
(1, 2), (3, 4), (5, 6), (7, 8, 9, 10), (11, 12, 13, 14), (15, 16, 17, 18), (19, 20, 21, 22), (23, 24, 25, 26), (27, 28), (29, 30)

The applied heuristic rules were as follows.

Rule 1: 'Shape defining bends are to be performed after non-shape defining bends'.
Rule 2: 'Bends in one branch of the connectivity graph are performed from the leaves towards the root'.
Rule 3: 'Shorter bends are to be performed before longer ones'.

5.3. *Obtained solutions*

A first solution was identified without need for backtracking due to early collision detection. The suggested sequence, (10-9-22-21-20-14-13-12-18-27-5-3-1-29-30-28-6-4-8-26-25-24-23-19-17-16-15-11-7), allows the production of the part with a minimum of required set-ups (22). All combinable bends are grouped into two operations. The respective obstructing bends are allocated after completion of these combined operations. A detailed breakdown of the penalty structure is presented in figure 20.

The generated solution can be improved as far as manipulation requirements are concerned. The complexity of the process plan for the machine operator could, e.g. be reduced by more systematic use of the symmetrical layout of the part. This factor was not covered by any of the applied heuristic rules, and could in consequence not be traced in the first identified solution.

Rule	# of Violations	Penalty Weight	Penalty
Bend Minimum	22	10	220
Combinable Bends	0	10	0
Continuous Series	9	3	27
Heurist. Rule 1	6	4	24
Heurist. Rule 2	1	2	2
Heurist. Rule 3	19	1	19
		Total	**292**

Figure 20. Sample part: penalty breakdown for initial sequence.

6. Conclusions

Two methods were described that allow the generation of realistic sequence proposals for further detailed analysis. The first method, based on precedence constraint solving, allows a significant reduction of the search domain by systematically eliminating all sequences that do not comply with the identified hard constraints, but is often not a sufficiently refined tool when conflicting heuristic rules are applied. Information that is not related to preferences for relative sequences cannot be represented in the developed procedure. When the complexity of the part description requires reasoning on continuous series of bends and/or bend combinability, or when heuristic rules can lead to accumulated preferences or preference conflicts, the second, TSP-based method proves to form a more reliable approach. A branch-and-bound search procedure, based on dynamic penalty calculations, was worked out to solve the reformulated sequencing problem. Case studies illustrate the early detection of near-optimum bending sequences by means of this TSP-based method.

Further research is being conducted related to the efficiency of the backtracking procedure for optimization purposes. The achieved results, however, already clearly illustrate that the complementary use of the described methods, combined with a well chosen set of heuristic rules, leads to an early identification of feasible and near-optimum solutions. The TSP-based method also offers the advantages of supporting a systematic comparison between process plan alternatives and offering a fast quantification in function of a manufacturability evaluation.

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