See discussions, stats, and author profiles for this publication at: [https://www.researchgate.net/publication/257745425](https://www.researchgate.net/publication/257745425_3D_Nesting_of_Complex_Shapes?enrichId=rgreq-29126b644684d9453ed24cb4e8556045-XXX&enrichSource=Y292ZXJQYWdlOzI1Nzc0NTQyNTtBUzoxMDMwNjQxMTE0MTk0MDdAMTQwMTU4MzgwNzYzNw%3D%3D&el=1_x_2&_esc=publicationCoverPdf)

[3D Nesting of Complex Shapes](https://www.researchgate.net/publication/257745425_3D_Nesting_of_Complex_Shapes?enrichId=rgreq-29126b644684d9453ed24cb4e8556045-XXX&enrichSource=Y292ZXJQYWdlOzI1Nzc0NTQyNTtBUzoxMDMwNjQxMTE0MTk0MDdAMTQwMTU4MzgwNzYzNw%3D%3D&el=1_x_3&_esc=publicationCoverPdf)

Article in Procedia CIRP · December 2012 DOI: 10.1016/j.procir.2012.07.006

All content following this page was uploaded by [Eric Lutters](https://www.researchgate.net/profile/Eric-Lutters?enrichId=rgreq-29126b644684d9453ed24cb4e8556045-XXX&enrichSource=Y292ZXJQYWdlOzI1Nzc0NTQyNTtBUzoxMDMwNjQxMTE0MTk0MDdAMTQwMTU4MzgwNzYzNw%3D%3D&el=1_x_10&_esc=publicationCoverPdf) on 01 June 2014.

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia CIRP 3 (2012) 26 - 31

45th CIRP Conference on Manufacturing Systems 2012

3D Nesting of Complex Shapes

E. Lutters^{a,*}, D. ten Dam^a, T. Faneker^a

a Laboratory of Design, Production and Management; University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands * Corresponding author. Tel.: +0-53 489 1197; fax: +0-53 489 3631; *E-mail address*: e.lutters@utwente.nl

Abstract

In application areas like Layered Manufacturing, there is a continuous struggle to optimize the filling degree of the production batches. This publication proposes an algorithm for the 3D nesting of complex shaped objects. The algorithm starts with the determination of the preferred orientation of a part, and uses a non-deterministic approach, closely related to the "Brazil Nut Effect" to do the actual nesting. The publication describes the algorithm and its steps, and describes case studies to demonstrate the validity of the approach.

© 2012 The Authors. Published by Elsevier B.V. Selection and/or peer-review under responsibility of Professor D. Mourtzis and Professor G. Chryssolouris. Professor G. Chryssolouris.

Keywords: Layered manufacturing; Nesting; Optimisation

1. Introduction

In many production processes, optimisation of material usage by means of nesting plays an important role. Traditionally, 2D nesting has been an aspect of sheet metal manufacturing. With the increasingly mature application of layered manufacturing, 3D nesting has become an essential part of the workflow. The reason for this is that the quality of the 3D nest not only influences the required amount of raw material and the quality of the individual products, but also has a direct impact on the throughput time of a production batch.

As a generic term, layered manufacturing (LM) represents additive production processes that construct 3D products by adding thin layers of material on top of each other and joining them together. The application of layered manufacturing ranges from rapid prototyping, via small batch part production to commercial one-of printing for consumers. Layered manufacturing encompasses a growing set of techniques, using specific materials and phenomena to construct the part.

All these processes have in common that the product volume is bound by the dimensions of the build volume or bin of the machine. Simultaneously, for many of the LM processes, the throughput time for producing one

bin is mainly dependent on the height of the build. This indicates the advantage to group as many products as possible per build. To improve the productivity and cost effectiveness of any LM process it is important to obtain a high degree of build volume utilization. To achieve this, the bin should be filled as efficiently as possible.

This nesting process is relatively straightforward for parts with simple geometries like boxes and cubes. However, the majority of parts that are produced by means of LM are complex shaped. Therefore a nesting approach is required that can cope with these complex shaped objects. Moreover, many existing approaches require a considerable amount of computation time, during which no 'valid' nesting solution is available.

This publication presents a novel method fill or 'nest' complex shaped parts into the available build volume. This method will unremittingly render a valid build, while continuously increasing the efficiency thereof.

2. Related literature

Nesting has been the topic of much research over the past years and still intrigues researchers today [1]. In general, most algorithms follow a three step procedure, being 'encoding', 'decoding' and 'evaluating' respectively.

Encoding The encoder generates different sequences that specify the order in which the parts need to be placed in the bin. For instance, consider three parts the encoder could then generate the sequence 3-1-2 meaning that part 3 needs to be placed first, then part 1 and finally part 2. In literature, many encoders are described [2], as there are many criteria (e.g. size, logistics, and priority) that influence the optimal sequence.

Decoding The decoder translates sequences generated by the encoder into a real world filled bin. A lot of research has focused on 2D decoders [3]. However, literature does not provide many solutions for 3D decoders. The reason why these decoders are important is because the performance of exact and heuristic methods is strongly influenced by the choice of a decoder [4], as it e.g. interprets the preferred orientation per part. Karabulut [5], Crainic [4], Maarouf [6] and Pisinger [7] describes various methods. The drawback of these methods is, however, the primary focus on nesting of boxes and cubes, as many of these approaches aim at e.g. the optimization of stacking packages. Ikonen [8] presents a method for complex shaped objects that uses a similar approach as Crainic.

Evaluation Once a sequence has been decoded the quality/fitness of that particular nest must be determined. Primarily, the goal of nesting is to generate a solution that has highest volume occupation. Literature provided a couple of criteria that can be used to assess the quality of a nest, including: stack height, utilized volume, number of parts or density distributions. The selected criterion depends on the goal of the algorithm.

3. Approach

The difficulty of finding a nest that has the highest volume occupation is that the solution space for a nesting problem is very large. Finding optimal solutions requires extreme computing times [2]. This is why conventional nesting algorithms use a three-stage approach. Instead of trying to find the optimal solution, the nesting problem is transformed into an optimisation problem using heuristic algorithms to generate (near-)optimal solutions. This approach reduces the size of the original solution space. However, the size of the remaining solution space is still of order (1).

$$
O(Z! * Y^Z) \tag{1}
$$

Here, Z is the number of parts and Y is the number of orientations per part (assumed to be equal for all parts). From this equation, it is apparent that the solution space can still be large, depending on the number of parts that must be nested and their rotational degree of freedom.

The nesting algorithm presented in this paper uses a two-stage approach:

- An orientation procedure is used to reduce the number of endorsed orientations per part.
- The actual nesting procedure focuses on generating a single nest that subsequently is improved.

This implies that the approach does not apply the time-consuming traditional tree-stage approach to aim at a (near-)optimal solution. Contrarily, the approach aspires to unremittingly render a valid nest. Moreover, the basic assumption that a nesting procedure should be deterministic is discharged.

4. Orientation

In order to reduce the number of orientations of a part, selection criteria are required that can be used to assess the appropriateness of different orientations. Here, two selection criteria are used that are combined with help of a multi-criteria evaluation method.

Fig. 1. Preferred nesting orientation

4.1. Preferred Nesting Orientation

The first criterion that will be used is the preferred nesting orientation. Consider the two parts in Figure 1a.

If one of these parts would be oriented as shown in Figure 1b, only one single part would fit in the bin, while both parts would fit if they were oriented as shown in Figure 1c. The idea behind this method is that the optimum nesting orientation is obtained when the volume of a parts 'Axis Aligned Bounding Box' (AABB) is minimal. Determining the preferred nesting orientation for complex shaped parts can be difficult. The above presented method is ideal for complex shaped parts; it may be redundant for box/cube-shaped parts.

4.2. Preferred Quality Orientation

The second criterion addresses the preferred quality orientation. Even though the goal of nesting is to fill the bin of the LM machine as efficiently as possible, the goal of LM is to produce high quality products. Pandey [9] demonstrates that the surface quality of a part depends on its orientation with respect to the build direction. This principle is shown in Figure 2 where the arrow indicates the build direction.

It demonstrates that surfaces that are not parallel or perpendicular to the build direction suffer from the socalled stair stepping effect. Lan [10] presented an algorithm that determines the amount of surface area that is affected by the stair stepping effect. Here, this algorithm is used to evaluate the surface quality.

Fig.2. AABB for a part in different orientations

4.3. Multi-Criteria Evaluation

To combine both evaluation criteria in one single scalar, multi-criteria evaluation is used. The general equation for a two criteria evaluation is given in (2)

$$
F = w_1 F_1(x) + w_2 F_2(y) \text{ where } w_2 = 1 - w_1 \quad (2)
$$

Here, w1 and w2 are user specified factors that affect the trade-off between both criteria in the obtained solution. $F_1(x)$ and $F_2(y)$ represent the first and second criterion respectively, thus F being a measure for the overall quality of an orientation.

Equation (2) can be rewritten into the following form:

$$
F = w_1 Q(b) + (1 - w_1)V(b)
$$
 (3)

Where Q(b) and V(b) are the optimization criteria for quality and bounding box volume that depend on the build direction b and are normalized with help of their maximum values $max(Q(b))$ and $max(V(b))$.

This approach allows for addressing additional criteria as well; in the context of this publication, however, the evaluation only focuses on nesting orientation and surface quality.

4.4. Part Orientations

The subsequent step in the process is to evaluate different part orientations. A simple but inefficient method would be to use a brute force approach were all possible orientations are evaluated. However, it is much more efficient to reduce the solution space in such a way that it only contains feasible solutions. To accomplish this it is important to look at the basics of both evaluation criteria.

In order to reduce the stair stepping effect, the parts must be oriented in such a way that the number of surface normals that are perpendicular or parallel to the build direction is maximal. This suggests that the optimal part orientation of a part with respect to quality points in the direction of one of the STL trianglenormals. This means that it is sufficient to only evaluate all the unique triangle-normals of the part.

These directions can also be used to determine the bounding boxes of a part. No unequivocal evidence has been found that these directions result in the absolute minimum volume bounding box of a part but this approach leads to adequate results.

5. Nesting

After the individual parts have been oriented they can be nested. As mentioned in section 2, the problem with existing approaches is that they focus on finding a (near) optimal solution. Based on the selected encoder this can be more or less deterministic. In any case, the calculation efforts required are often extremely high, especially in realising that a small increase in achieved quality involves disproportionate computational efforts. Although for some industries such small improvements may be worth the effort, for layered manufacturing the reduction in material costs does not outweigh the increased costs attributed to the extra time required to generate this nest.

Consequently, a different approach may be suitable for layered manufacturing. This approach not necessarily results in a (near) optimal solution as long as it is fast and produces a solution that is acceptable. Moreover, it is important that at any point in the process a valid solution can be provided. This means that a compromise between available time for nesting and quality of the nest can always be made, thus avoiding either standstill of the machines and longer throughput times.

5.1. Alternative Nesting Procedure

An intriguing method for 2D nesting is presented by Jia [11]. Unlike the traditional methods, it first generates an initial nest and tries to improve its packing density by applying high frequency vibrating motions. These vibrations evoke the so-called 'Brazil Nut Effect' [12]. This effect can best be explained with help of an example. Consider two granulates whose grains differ in size. These two granulates are placed on top of each other in a confined space where the granulate with the largest grains is placed at the bottom as shown Figure 3.

Fig.3. Brazil Nut Effect

A vibrating motion is applied to the bin in Figure 3a. These vibrations allow smaller spheres to pass through the cavities between the larger spheres (Figure 3b). If vibrations are applied long enough, size segregation will occur (Figure 3c). To achieve a workable solution, the idea is to apply just enough vibrations so that the smaller parts will fill the cavities between the larger parts, with a result as shown in Figure 3b.

5.2. Implementation

In order to adequately implement the 'Brazil Nut Effect' for the nesting of complex shapes in 3D bins for layered manufacturing, the parts to be nested are required to show some 'dynamic behaviour'. For example, overlap between the parts needs to be avoided during the vibrating motions. The problem with overlap calculations, however, is that they become more and more computationally expensive once the complexity of the part increases. Using the original complex shaped parts in dynamic modelling would yield extremely high computing times. As a solution, a part can be approximated with primitive bounding volumes. Figure 4 displays a part that is approximated with a sphere, AABB, oriented bounding box (OBB), k-dop and a convex hull. The tightness and complexity of the fit increases from left to right, so the cost of overlap calculations increases from left to right as well.

The nesting algorithm is based on the notion that nesting speed can be increased at the cost of bounding volume accuracy. The algorithm starts by approximating the parts with bounding spheres but gradually switches to more tightly fitting bounding volumes. This approach results in fast, yet low quality, nests. The quality of this nest will increase as time progresses. In other words nest quality will depend on the time that is available for nesting. This is advantageous because when this nesting process is incorporated into a supply chain the nesting time can be aligned with the demands of the workflow.

Based on this notion, a nesting algorithm has been developed. The flowchart in figure 5 depicts the steps in the algorithm. The subsequent sections describe the blocks in more detail.

Fig.5.Flowchart of nesting algorithm

Determine Maximum Nesting Level/Time: The first step in this process is to define the available time for nesting. If more time is available it will lead to a higher quality, this although the nesting process can be stopped at any time and a valid nest is obtained. It is also possible to specify the maximum nesting level. Increasing the bounding volume level beyond OBB for a batch that only consists out of boxes or cubes would needlessly increase computing times.

Calculate and Sort Bounding Spheres: The unique STLs within a batch are read into the nesting algorithm. This data is used to calculate the minimum volume bounding sphere for every part. Once this has been completed, the part quantities can be specified that are applicable for that bin. Finally, the spheres are sorted based on their radii in a descending order. The reason for this is that the larger spheres must be located at the bottom for the Brazil Nut Effect to be effective. Also, it is more likely that the larger spheres still fit into the bin when they are placed first instead of last.

Alternative Nesting Procedure: The use of bounding spheres may result in problems; when large parts are approximated with a bounding sphere it can result in a sphere that no longer fits into the bin. These parts must be nested with help of an alternative nesting routine. The parts are approximated with an OBB instead of a bounding sphere. Another problem that may occur when parts are approximated with a bounding volume is that the volume of the bounding sphere is much larger than the actual volume of the part. Here, the part is approximated with low level sphere trees.

Generate Initial Nest: The bin filling process is modelled dynamically. A 'rain model' is used where Fig.4. Different bounding volumes parts start at the top and are given a random x and y position and initial random velocities in x, y and z direction. This allows the parts to settle.

Bin Vibrations: Simulated vibrations are applied to the bin in order to try to improve the quality of a nest. In order to determine if the quality has improved, a performance indicator is required. Here, the maximum bin height (MBH) was selected, representing build time.

Void Filling Procedure: This procedure checks the obtained nest for cavities and tries to fill them with parts from the top layer of the nest.

Determine Available Nesting Time/Level: The next step is to determine if the available nesting time, which has been specified at the beginning, has expired or that the desired nesting level has been obtained. If this is the case, the obtained nest is sent to the 'Export Nest to Machine' procedure where it is prepared for actual production. Otherwise the nest is sent to the 'Next Level Bounding Volume' procedure.

Next Level Bounding Volume: The second level bounding volume applied is an OBB (oriented bounding box); however, only large parts are approximated with OBB trees. The reason for this is that even though small parts could be better approximated with an OBB tree the volume reduction that is obtained is small compared to the extra computations that are required to determine overlap. Bounding volume refinement can continue until the bounding volume is approximately equal to the actual volume of the part.

Export Nest to Machine: In this step, the obtained nest is prepared for actual production. The bounding volumes are exchanged with the real STLs, the bin is sliced and tool paths are created. Finally, the data is sent to the printing machine and the parts are manufactured.

6. Results

6.1. Orientation

 Because of the fact that the orientation algorithm uses a multi-criteria hybrid approach, different compromises between the preferred nesting orientation (w_1) and surface quality $(w_2, \text{ or } 1 - w_1)$ will lead to different orientations. In order to demonstrate this, three different values for w_1 have been employed; $w_1 = 0.0$, 0.5 and 1,0. These values respectively represent an orientation that is optimal in terms of nesting, a balance between both criteria and optimal in terms of quality.

6.2. Nesting

In order to test the nesting algorithm two different test batches were randomly selected from actual (commercial) builds. The results are presented according to the steps in the algorithm. The dimensions of the test bin used are 300 x 300 x 300 mm (L x W x H).

Calculate and Sort Bounding Spheres: After calculating the radius of the bounding sphere, the parts are sorted according to radius.

Generate Initial Nest: The bounding spheres are added to the bin. The results are shown in Figure 6. The bin in Set 1 suffers from overfilling. Set 2 show that the initially obtained nest is clearly not optimal. Some of spheres are stacked in an unstable situation.

Bin Vibrations (1): In order to improve the quality of the initial nests, vibrations are applied. The effect of different vibration parameters was determined with help of experiments. Variables included are: vibration type, phase shift, amplitude, frequency and direction. From the experiments, the vibration settings were selected that resulted in the lowest MBH. The bins shown in Figure 7 were obtained with these settings.

Next Level Bounding Volume: The spheres are replaced with the next level bounding volume, which are OBBs. (see Figure 8). After the transition, the obtained bin configuration is allowed to settle to a new equilibrium. Results are shown in Figure 9.

Bin Vibrations (2): The bins are again subjected to vibrations in order to improve the quality of a bin. The results are shown in Figure 10. Again, the results show a reduction in MBH. This demonstrates that the Brazil Nut Effect indeed improves the quality of a nest.

Export Nest to Machine: Once the desired nesting level has been obtained or the time for nesting has expired, the bin is prepared for manufacturing. This means that the current bounding volumes are exchanged with the actual STL's, slices and tool paths must be created and the data must be uploaded to the printer. Figure 11 shows the bins that are obtained after the bounding volumes have been replaced with the STLs.

6.3. Performance

The prototype implementations have been realised with a strong focus on functionality and proof-ofconcept. As a result, parts of the implementation have been realised in Matlab, using readily available, but generic, procedures. Consequently, attention for calculation times in efficiency of implementation has not gained high priority. Nevertheless, the results have been quite promising. For example, in terms of bin filling (or MBH) the algorithm showed results that are comparable to industry best practices. As mentioned, calculation times of the prototype software are not realistic, because of the proof-of-concept character of it. Nevertheless, the calculation times encountered are quite acceptable. For the orientation procedure, calculation times for Set 1 are a few minutes. The first phase in the nesting procedure also takes but a few minutes. With increased quality of the implementation and adequate computing power, computation times can be reduced considerably.

7. Concluding remarks

With the case studies described in section 6, it is demonstrated that the presented nesting algorithm does provide valid nests, and that the nesting converges to improved solutions. With help of the weight factor it is possible to emphasize the importance of a certain evaluation criterion. Here, two evaluation criteria were considered; nesting and surface quality. However, the multi-criteria evaluation method can easily be extended with other factors as well. The nesting results clearly show that the Brazil Nut Effect can be used to improve the quality of a nest. Even though the nesting prototype is in its infancy and some of the steps in the algorithm can certainly be improved, it already shows promising results. Furthermore, the algorithm is able to generate nests where the quality of the nest depends on the time that is available for nesting. This allows the user to set the nesting time to the demands of the workflow when it is implemented into a supply chain.

References

[View publication stats](https://www.researchgate.net/publication/257745425)

- [1] Nee, A.Y.C., Seow, K.W., Long S.L., 1986. Designing algorithm for nesting irregular shapes with and without boundary constraints, Annals of the CIRP, 35/1:107-110
- [2] Cagan, J., Shimada, K., Yin, S., 2002, A survey of computational approaches to three-dimensional layout problems, Computer-Aided Design, 34/8:597-611.
- [3] Hopper, E., Turton, B.C.H., 2001, A review of the application of meta-heuristic algorithms to 2D strip packing problems, Artificial Intelligence Review, 16: 257-300.
- [4] Crainic, T.G., Perboli, G., Tadei, R., 2008, "Extreme-Point-based Heuristics for the Three-Dimensional Bin Packing problem". In: Informs Journal on computing, Vol. 20, pp. 368–384
- [5] Karabulut, K., Inceoglu, M.M., 2005, A hybrid genetic algorithm for packing in 3D with deepest bottom left with fill method, In: Advances in Information Systems, Vol. 3261/2005, 441-450
- [6] Maarouf, W.F., Barber, A.M., Owayjan, M.J.A., 2008, Innovations and advanced techniques in systems, computing sciences and software engineering, In: Innovations and Advanced Techniques in Systems, Computing Sciences and Software Engineering, pp. 342-345.
- [7] Pisinger, D., 2002, Heuristics for the container loading problem, In: European journal of operational research, 141/2: 382-392
- [8] Ikonen, I., Biles, W.E., Kumar, A., Ragade, R.K., 1996, Proceedings of the first online workshop on soft computing.
- [9] Pandey, P.M., Reddy, N.V., Dhande, S.G., 2003, Improvement of surface finish by staircase machining in fused deposition modeling, In: Journal of materials processing technology, 132: 323-331
- [10] Lan, P.-T, Chou, S.-Y, Chen, L,-L, Gemmill, D., 1997, Determining fabrication orientations for rapid prototyping with stereolithography apparatus, Computer-Aided Design, 29/1: 53- 62
- [11] Jia, X., Williams, R.A., 2001, A packing algorithm for particles of arbitrary shapes, In: Powder Technology, 120: 175-186
- [12] Rosato, A., Strandburg, K.J., Prinz, F., Swendsen, R.H., 1987, Why the Brazil nuts are on top: Size segregration of particulate matter by shaking, Phys. Rev. Lett., 58/10: 1038-1040

Fig.6. Initial nest Fig.7. MBH after vibration

Fig.8.Next level bounding volume Fig.9.Settled OBB bins

Fig.10. Vibrated OBB bins Fig.11. Conversion to STL bins