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Technical Note

On the Role of Geometry in Mechanical Design

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A complete design usually specifies a mechanical system in terms of component parts and assembly relationships. Each part has a fully defined nominal or ideal form and well defined material properties. Tolerances are used to permit variations in the form and properties of the components, and are used also to permit variations in the assembly relationships. Thus the geometry and material properties of the system and all of its pieces are fully defined (at least in principle). Henceforth we shall focus on geometry and, for reasons that will become evident, will not deal with materials despite their obvious importance.

Mechanical systems specified in the manner just described meet functional specifications that appeared initially as design goals. The process of design can be thought of as "generating the geometry"—the breakdown into components with coarsely specified geometry, and then the detailed specification of the component forms and fitting relationships. Design seems to proceed through *simultaneous refinement* of geometry and function [1]. An important line of design research seeks scientific models for this refinement process and systematic procedures for improving and perhaps automating it.

At present we have tools for dealing with two widely separated stages of the refinement process.

- For single parts, function is usually specified through loads on pieces of surface (e.g. a force distribution over a support surface, a flow rate through an orifice, a radiation pattern over a cooling fin); specification of the solid material that provides a carrier for the pieces of surface may be viewed as a constrained shape optimization process.
- At the higher level of "unit functionality," where one deals with springs, motors, gear boxes, heat exchangers, and the like, geometry usually is abstracted into real numbers if acknowledged at all, and function is cast in terms of ordinary differential or algebraic equations (for heat flow, motor torque as a function of field current, and so forth). Systems of such equations describe the composite functionalism of networks of functional units.

There is a big gap between these "islands of understanding," and intermediate stages of abstraction are needed which acknowledge the partial geometry and spatial arrangement topology of subassemblies. Broadly speaking, geometry is faring badly in contemporary design research; many investigators either "sweep it under the carpet" or deal with it syntactically, e.g. through "features" defined in ad hoc ways. Clearly we need more systematic ways to address the relationship between geometry and function, and we suggest below some initial steps toward this goal.

Energy Exchange as a Mechanism for Modeling Mechanical Function

Mechanical artifacts interact with their environments through spatially distributed energy exchanges, and we argue below that mechanical functionalism can be modeled in terms of these exchanges. The initial cast of the argument draws heavily on seminal work by Henry Paynter [2].

We shall regard mechanical artifacts as *systems* that range from single solids or fluid streams, which usually are the lowest level of natural system that exhibit important properties of mechanics, to complex assemblies of solids and streams. A closed boundary, which may be physical or conceptual, is a distinguishing characteristic of a system: the system lies within (and partially in) the boundary, the environment lies outside, and interaction occurs

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through the boundary. We distinguish the following:

S : the physical system under discussion;
δS : the boundary of S;
V : a spatial region containing S whose complement is the environment;
δV : the boundary of V.

S may coincide with V, and δS and δV are closed surfaces (usually 2-mainfolds) in E^3. We distinguish S from V because S may be partially or wholly unknown (recall that this note is about design) but boundable by a known V.

The principle of continuity of energy applies at all levels of system abstraction. If no energy is generated by the system, then

$$\int_{\delta V} \mathbf{P} \cdot \mathbf{n}\, d(\delta V) = \int_V \frac{\partial \varepsilon}{\partial t}\, dV + \int_V g\, dV. \tag{1}$$

The surface integral on the left describes the total energy flux (instantaneous power) through the boundary; **P** is a generalized Poynting vector describing the instantaneous rate at which energy is transported per unit area, and **n** is the normal at a point in the boundary δV. On the right, $\partial\varepsilon/\partial t$ is the (volumetric) density of energy stored in the system, and g is the rate of energy loss or dissipation.

A system interacts with its environment by exchanging energy through its physical boundary: for example, by radiating energy stored in the system over a portion of its area, or by providing support to an external mating part and thereby inducing storage of deformation energy in the system. The subsets of the physical boundary over which such exchanges occur will be called (following Paynter) *energy ports*. If s_i is the physical boundary subset ('piece of surface') associated with the i^{th} port, then

$$\sum_i \int_{s_i} \mathbf{P} \cdot \mathbf{n}\, ds_i = \int_V \frac{\partial \varepsilon}{\partial t}\, dV + \int_V g\, dV \tag{2a}$$

where

$$\bigcup_i s_i \subseteq \delta S. \tag{2b}$$

Thus the total energy flux through the boundary is a sum of signed fluxes through the ports. We note that a boundary subset s_i may belong to several ports, and that body forces, such as those induced by gravitational and magnetic fields, may be accomodated by taking δS as the associated port.

Geometrical and Functional Refinement in the Limit

The left side of Eq. (2a) specifies energy exchanges through the system's ports and requires that the flux vector(s) and port geometries be known. The terms on the right cover internal energy (re)distribution and/or dissipation. The physical effects implied by these terms depend on the energy regime(s) and the geometry of the system; there may be rigid body motion, elastic or plastic deformation, temperature redistribution, and so forth. Mathematical evaluation requires the solution of 3-D boundary- and/or initial-value problems.

Very marked simplifications ensue if one assumes that 1) the ports are spatially localized and idealized so that the integrals on the left of Eq. (2a) may be evaluated individually to yield terms P_i, and 2) internal energy storage and dissipation are similarly localized in disjoint discrete regions, thereby permitting the right-hand integrals to be decomposed into sums of local integrals which may be evaluated individually. With these assumptions, Eq. (2a) may be rewritten

$$\sum_i P_i = \sum_j \frac{\partial E_j}{\partial t} + \sum_k G_k \tag{3}$$

where P_i is the power through the i^{th} discrete port, E_j is the instantaneous energy stored in the j^{th} discrete region, and G_k is the dissipation rate in the k^{th} discrete region. A limiting form of this refinement (or discretization, or—in Paynter's terminology—reticulation) is a "Dirac-delta limit" wherein the ports shrink to spots of zero area and the volumetric regions shrink to point masses, idealized resistors, and the like.

Equation (3) is the basis for Paynter's energy bond diagrams, or bond graphs. It describes a system that may transfer, transform, store, and dissipate energy through elements whose geometry has been refined into a few real numbers—the spatial positions of the discrete ports and lumped regions (which generally are not carried in bond-graph representations), and integral characterizations of the discrete ports and regions (for example the "value," in kilograms, of a point mass). This higher view enables one to analyze the dynamics of the idealized (discretized) system, but one can *deduce* little about the geometry of feasible distributed (i.e., real) systems from such analyses; essentially all geometry must be *induced*. Apparently we have gone too far, i.e., have thrown away too much geometry.

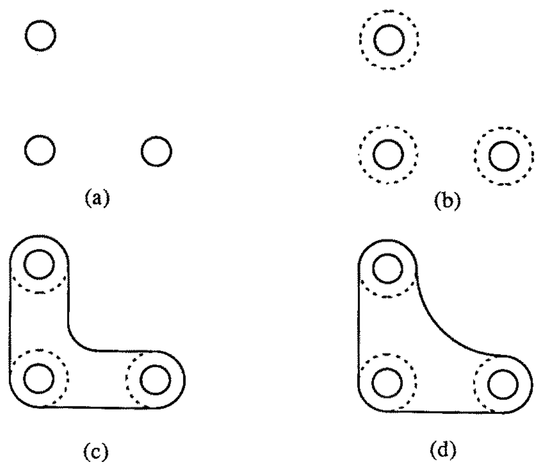

Fig. 1. Design of a simple bracket.

Toward an Appropriate Role for Geometry

We would like to step back from the limiting refinement just discussed, where all notions of form have been lost, and include in the problem some continuous geometry—but not the full-blown field problem covered by Eq. (1) unless this is unavoidable. We shall suggest below three principles governing the interaction of form and function that we believe will yield geometrically well defined (but not necessarily optimum) designs. A simple but common example drawn from practice—design of a bracket—will motivate the discussion (Fig. 1).

The design begins with three holes of known diameter and configuration that are to be carried by an unknown solid (Fig. 1a); these mate with other parts (two screws and a pivot pin). Bosses are created to contain the holes (Fig. 1b) because of concern about interference with other components passing between the holes. Finally the holes and bosses are bound together into a single part as in Figs. 1c and 1d, with the final shape being governed by criteria for clearance, strength, weight, and aesthetic and manufacturing simplicity.

Two simple but important inferences may be drawn from the example. Firstly, the initial holes (plus some implied constraint surfaces in the third dimension) are the bracket's energy ports; they are fully specified geometrically and specify by implication what the bracket is to do—maintain the relative position of ports whose geometry admits rotational motion. In principle the associated energy regimes (force, torque : elasticity) can be fully specified as well, but in practice they are often only implied or "understood." Secondly, the remaining geometry is discretionary but constrained by requirements that the holes be bound into a connected solid, that

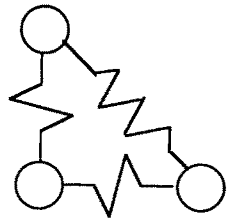

Fig. 2. Position-fixing character of the bracket.

the solid not interfere with other components, and so forth. We note that, at the single-component level of the bracket, shape optimization usually *does* require solution of the full 3-D field problem covered by Eq. (2a).

From this example and others we induce:

Principle 1. A system's "function" is determined by its energy ports, which are generally subsets of its physical boundary, and the energy regimes operating on those ports; both should be fully defined. The remaining geometry of the system is discretionary provided that 1) it admits at least one physical realization of the system that satisfies the port specifications, and 2) other external constraints, e.g. on overall size, are met.

Principle 2. Energy exchanges within a system always may be represented independently of geometry, e.g. via bond graphs.

Figure 2 shows the position-fixing capabilities of the bracket represented (nonuniquely) by ideal springs attached to the locally rigid ports. This representation of the bracket's partial functionalism assumes ideally elastic behavior, and this assumption should be checked, e.g. by finite-element analysis, as the bracket's final shape is being determined.

Figure 3 shows a slightly more complicated system—an indicator that senses pressure via an orifice (port) of known geometry, and displaces a rotary indicator correspondingly. The output indicator is a port because we require that it be able

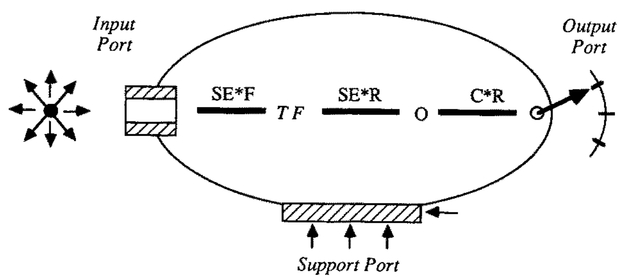

Fig. 3. A pressure measuring system.

to do work on the environment, e.g. overcome specified restraining torques over a defined range of travel, and hence its geometry must be defined. The system also has a third support port. The system's primary function is represented internally by a pressure/torque transformer and a rotary spring which are shown as bond graph elements in the style of Ulrich and Seering [3], but *this representation is not unique*; it may be replaced with other, arbitrarily elaborate arrangements of idealized elements having the same input/output functionalism plus other paths that terminate internally.

Equation (4) provides the rationale for Principle 2. The essential idea is that the port

$$\sum_i \int_{s_i} \mathbf{P} \cdot \mathbf{n} \, ds_i = \sum_j \frac{\partial E_j}{\partial t} + \sum_k G_k \qquad (4)$$

flow on the left of Eq. (2a) may be handled internally (the right-hand integrals in Eq. (2a)) in many ways. If we are assured by Principle 1, or simply assume, that internal solutions exist, then we may reticulate the internal geometry and deal with integral quantities as in Eq. (3).

Principle 3. Principles 1 and 2 must hold for all subsystems defined on combinatorial decompositions of a system.

Principle 3 provides means for the simultaneous refinement of geometry and function. It enables complicated systems to be decomposed recursively into functional subsystems provided that one defines the ports as one proceeds. The limiting combinatorial refinement is single parts, and at this level one must solve the field problem of Eq. (2a) to obtain complete geometric specifications.

Concluding Remarks

The thoughts above are aimed at finding means to establish for geometry an appropriate formal role in a theory of mechanical design. It seems obvious to us that geometry should have such a role, but the work needed to establish it has barely begun.

Epilogue—Remarks on Features

This work grew out of a several-month effort to characterize geometric features in a formal manner—an effort that largely failed. The effort was motivated by the fact that mechanical design and manufacturing are often discussed and done in terms of "features," but there are no agreed views on what features "are" or "do" [4]. (Slots, ribs, webs, and shafts, are typical features; all involve geometry in one way or another.)

We began with a conjecture: A geometric feature may be defined as a geometric idealization of a port for energy exchange in a defined regime. (This notion is appealing because it implies that a system's feature-set specifies all of the geometry needed to define the system's interactions with its environment; the remaining geometry is determined by constraints and optimization.) We then proceeded to show that the conjecture is formally consistent in design, manufacturing, and inspection applications. In machining, for example, geometric features may be associated with the boundary of the removed material; the energetic process is machining itself, whose dynamics are reasonably well understood in a macroscopic sense. Clamping features may be defined primarily through elastic energy storage, inspection features through the energetic exchange involved in the measurement process, etc. But as our explanations grew increasingly contrived and our difficulties with solid and other non-surface features mounted, we began to sense that features could not be defined in any universal system other than a purely syntactic system.

Currently we believe that features are simply information structures that represent, often in parametric form, known solutions to local problems. While a syntactic structure can be imposed on them, their underlying semantics can vary widely and need not involve particular kinds of geometry, or indeed any geometry at all. However, if a feature is to be used properly, a feature-context must be supplied—the technical conditions and criteria that led to the solution the feature represents. Given the feature-context (e.g., as domain knowledge in a designer's head) *and* appropriate reasoning power to adapt the solution to the current problem, features can be very effective; their popularity among human designers attests to this.

Recent work by Duffey and Dixon [5] illustrates that features can be used in automatic design when feature-contexts and appropriate reasoning power are provided. (The handling of features by Duffey and Dixon seems ad hoc, but "ad-hocery" may be intrinsic if our current permissive view of features is correct.) Features can be dangerous when used without their contexts and appropriate reasoning power, as nonsense designs produced by certain automatic design systems illustrate.

Finally, we wish to point out that the characterization of features as "known solutions to local problems" places strong constraints on schemes for combining features to make new features. A feature combination makes sense only if it can be shown to be a valid solution to a well defined local problem. But even determining the domain of the combination problem as a function of its component domains may prove very difficult.

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